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A booklet designed for elementary school students to be used independently from and in addition to the regular classroom curriculum in mathematics is presented. The fifth-or sixth-grade student is given practice in working problems in subtraction and division. Short cuts and hints are presented through examples, sample problems, and answers. The teacher identifies potential users, orients them to the area to be explored, and stimulates and maintains their interest in sustained independent effort.

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INDEPENDENT LEARNING BOOKLET

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MENTAL ARITHMETIC
IN SUBTRACTION AND DIVISION

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DEPARTMENT OF EDUCATION • SAN DIEGO COUNTY

INDEPENDENT LEARNING BOOKLET

Mental Arithmetic In Subtraction and Division

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TO THE TEACHER

This Independent Learning booklet is designed to help you open new avenues of learning for students who show a special interest in mathematics and a readiness to study on their own. Such students will be able to work directly from printed materials, using abstract ideas to follow a line of mathematical reasoning. Occasionally, of course, even the ablest of them will need your help to get over some of the rough spots, but your main concern in connection with the booklet will be to identify students who can benefit from using it, to orient them to the area to be explored, and to stimulate and maintain their interest in sustained independent effort.

Since the topic of the booklet trends outward from the regular curriculum, the challenge to do this independent work should arise from students' own searching in the field or from new interests that you have succeeded in awakening. Students will profit most if they elect voluntarily to do the work rather than receiving it as an assignment or as a means of earning extra credit.

The material is so organized that students may progress at their own best pace. They should be encouraged to move ahead in discovering for themselves meanings and approaches in mathematics beyond those usually taught in the classroom, but there should be no requirement to complete parts or all of the booklet, or to follow a set schedule.

While serious effort will be necessary, students should find the work enjoyable because of their interest in the subject. They should gain a real sense of satisfaction through working individually or with other students in independently achieving understanding, overcoming temporary difficulties, and evaluating their own progress. Your encouragement and occasional direct help will of course be needed.

This booklet obviously is not aimed at developing complete understanding of an area of mathematics. Its purpose is rather to provide an introduction, or an approach novel to the student, which will create new interests and the desire to go on from here to further study in other sources.

A brief teacher's guide has been prepared to assist you in using this booklet with your students. Copies of the guide are distributed to your district office by the Department of Education, San Diego County. Additional copies are always available.

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INTRODUCTION

The purpose of this booklet is to show you some new and interesting ways to do subtraction and division. These new methods will enable you to do more of your arithmetic work in your head and less of it on paper. If you like arithmetic you will find these methods not only efficient but fun to do.

You will be learning directly from this booklet, so follow the explanations carefully. Your teacher will help you if you really get "stuck," but don't ask for help until you have worked through a section several times. You will feel more pleased with your accomplishments if you have achieved them on your own.

A word of advice: The booklet is divided into several parts. You will have smooth sailing if you do enough practice in each part to become skillful in using the ideas it develops before you go on to the following part.

While you should have pencil and paper ready to use when you need them, do as much of the work as possible without writing. Since other boys and girls will be using this same material, please do not write anything in the booklet itself.

Recalling Some Ideas

To get off to a good start, let's first review some vocabulary and ideas with which you are already acquainted. Having them clearly in mind is important to your success with the new ways of working introduced in this booklet.

Digits and Numerals

The symbols **0, 1, 2, 3, 4, 5, 6, 7, 8** and **9** are called **digits.** We use these ten digits in writing all the **numerals** in the decimal place-value system of notation. Just as words stand for ideas, so numerals stand for number ideas. The numeral **340** stands for the number idea we call "three hundred forty." The digits of 340 (**3, 4,** and **0**) could be used to write different numerals, such as 304, 430, 403, etc., each of which is a distinct number idea.

When a digit stands alone, for example a 2, a 5, or an 8, we may speak of it as a digit or as a numeral.

Integers

Integers are whole numbers rather than fractional numbers. The following numerals stand for integers:

6 53 108 1790

The following numerals stand for fractions:

$\frac{1}{2}$ $\frac{5}{8}$ .1 .25

Sometimes we speak of the part of a number that is an integer. The 9 of 9.1 and the 8 of 8.6 are examples.

Subtraction

Subtraction is the inverse, that is, the opposite or undoing, of addition. You know that when you add, you **combine** quantities. You can see the relationship of addition to subtraction in this illustration: $2 + 3 = 5$ and $3 + 2 = 5$, therefore $5 - 3 = 2$ and $5 - 2 = 3$.

Minuend

The **minuend** in subtraction is the quantity from which another quantity is to be subtracted. For example, 5 is the minuend in the subtractions above.

Subtrahend

The **subtrahend** is the quantity which is to be subtracted from another quantity. For example, in $5 - 2$ the subtrahend is 2.

Difference or Remainder

The result of subtracting is called the **difference** or **remainder.**

For example:

$$\begin{array}{r} 5 \\ -\,2 \\ \hline 3 \end{array}$$

5 Minuend
−2 Subtrahend
3 Difference or Remainder

Minuend − Subtrahend = Difference or Remainder

Division

Division is the inverse of multiplication. You know that when you multiply, you combine equal quantities to find a total, called the product. For example, you combine 4 threes to get the product 12. Since 4 threes equal 12, you can take 4 threes out of 12. We say that $4 \times 3 = 12$, therefore $12 \div 3 = 4$.

Dividend — The **dividend** is the quantity which is to be divided.

Divisor — The **divisor** is the quantity by which the dividend is divided.

Quotient — The **quotient** is the quantity resulting from the division.

For example:

$3\overline{)12}\quad 4$

and

$12 \div 3 = 4$

Dividend	12
Divisor	3
Quotient	4

$$\text{Divisor}\overline{)\text{Dividend}}\quad \text{Quotient}$$

Dividend ÷ Divisor = Quotient

$8\overline{)272}$

Steps in Division—Follow the steps with the example at the left to see how the division process can be performed. Because of the way the decimal place-value system works, we can begin at the left and work toward the right. We divide the larger place values first and work with part of the dividend at a time. We call this a **partial dividend.**

Step 1

$8\overline{)272}$

1. Select the first partial dividend. Is 8 contained in 2 at least once? No. In 27? Yes, therefore 27 is our first partial dividend.

Step 2

$8\overline{)272}\quad 3$

2. Divide 27 by 8 to get the first quotient digit. How many 8's in 27? $3 \times 8 = 24$, $4 \times 8 = 32$; 3 must be correct. Write the 3 in tens' place because the 27 stands for 27 tens.

Step 3

$8\overline{)272}\quad 3$

24

3. Multiply and compare. $3 \times 8 = 24$. Is 24 no larger than 27?

Step 4

```
      3
   ------
8 ) 2 7 2
    2 4
    ---
      3
```

4. Subtract 24 from 27. Compare to see that the difference (3) is smaller than the divisor (8).

Step 5

```
      3
   ------
8 ) 2 7 2
    2 4
    -----
      3 2
```

5. Three tens and 2 ones remain to be divided. Bring down the 2 ones to make the next partial dividend, 32.

```
      3 4
   ------
8 ) 2 7 2
    2 4
    -----
      3 2
      3 2
      ---
```

Now do these steps:
Divide 32 by 8.
Write the 4 in ones' place.
Multiply and compare.
Subtract and compare.
There is no more to be divided; the division is completed.

```
        2 5 1
   ----------
3 2 ) 8 0 3 5
      6 4
      -----
      1 6 3
      1 6 0
      -----
          3 5
          3 2
          ---
            3
```

Now study this example, which has a two-digit divisor. The first partial dividend is 80. The 8 is in thousands' place and the 0 is in hundreds' place; there are 80 hundreds. Because of the way our decimal place-value system works, when we are performing the division we need only to think, How many 32's are there in 80? The next partial dividend is 163. The last partial dividend is 35 since we are not "carrying out" to decimal fractions. We have 3 left over, which we call a remainder. The quotient could be written $251\frac{3}{32}$.

In the example above the partial dividends 80, 163, and 35 are also minuends, while 64, 160, and 32 are subtrahends. We obtained these subtrahends by multiplying the divisor by the appropriate digit of the quotient:
$2 \times 32 = 64$, $5 \times 32 = 160$, and $1 \times 32 = 32$.

Checking Division—Division can be checked by multiplication.

Division

$$\begin{array}{r} 251 \\ 32\overline{)8035} \\ 64 \\ \hline 163 \\ 160 \\ \hline 35 \\ 32 \\ \hline 3 \end{array}$$

Multiplication

$$\begin{array}{r} 32 \\ \times 251 \\ \hline 32 \\ 160 \\ 64 \\ \hline 8032 \\ + \quad 3 \\ \hline 8035 \end{array}$$

When we divide we start with the largest place values and work from left to right. When we multiply we start with the smaller place values and work from right to left. (Other ways of working are possible; these are the ones ordinarily used.) Can you see the partial dividends of the division above in the partial products of the multiplication?

Estimating and Approximating

When we **estimate** in arithmetic we use our best judgment about an answer without working it out exactly, that is, we **approximate** the answer, or approach it closely. Estimating and approximating help us judge the reasonableness of the results of calculation.

Rounding

Rounding numbers is helpful in estimating; when we "round" we approximate the value of the number. For example, if we want to estimate the quotient of $32.13 \div 5.1$ we can think of 30 divided by 5 and estimate that the quotient will be close to 6 (the exact quotient is 6.3). You may want to study the material on rounding numbers in your textbook.

Do you have each point above clearly in mind? If so, you are ready to learn the new ideas which are introduced in this booklet. You can refer to this part of your booklet again if it will help you as you work with the following sections.

MENTAL ARITHMETIC IN SUBTRACTION AND DIVISION

PART I

Adding Something New to Subtraction

"There's more than one way to skin a cat." This old saying applies to many of the ways that we work with numbers. You have known how to subtract for a long time, but you probably have "skinned the cat" the same way every time.

Let's take a look at the way you would probably work this example: 765 – 379. As you read the explanation, it may help you to point to each numeral in the example at the left when it is mentioned.

Step 1

$$\begin{array}{r} {\scriptstyle 5\ 15} \\ 7\ \not{6}\ \not{5} \\ 3\ 7\ 9 \\ \hline 6 \end{array}$$

Step 2

$$\begin{array}{r} {\scriptstyle 6\ 15\ 15} \\ 7\ \not{6}\ \not{5} \\ 3\ 7\ 9 \\ \hline 8\ 6 \end{array}$$

Step 3

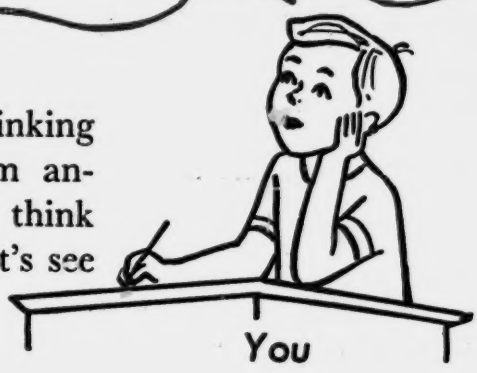
$$\begin{array}{r} {\scriptstyle 6\ 15\ 15} \\ 7\ \not{6}\ \not{5} \\ 3\ 7\ 9 \\ \hline 3\ 8\ 6 \end{array}$$

1. I can't subtract 9 ones from 5 ones, so I change one of the 6 tens to 10 ones, add these to the 5 ones, and take away 9 ones from 15 ones.

2. I can't subtract 7 tens from the 5 tens remaining, so I change one of the 7 hundreds to 10 tens and take away 7 tens from 15 tens.

3. Next I take away 3 hundreds from the remaining 6 hundreds.

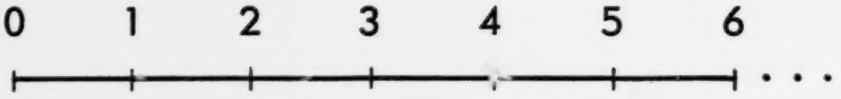
Notice that we have been thinking "taking away" one number from another. There is another way to think of these number relationships. Let's see if you can discover it.



How would you think in solving the following problem? "I have two dollars. How much more money do I need in order to have five dollars with which to buy a sweater?" Did you think, "Two dollars **plus** what equals five dollars; I need three dollars"? Or did you think, "Five dollars **minus** two dollars equals three dollars"? In either way of thinking you arrive at the difference of three dollars, don't you?

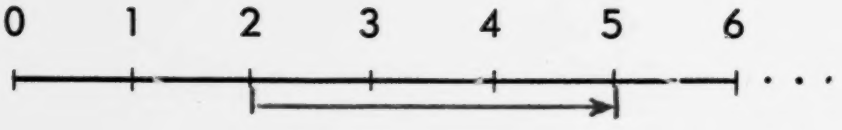
Another way to illustrate the relationship between addition and subtraction is to use a number line. The one we use here is marked for part of its length; the three dots at each end indicate that you can make and mark a number line in both directions as far as you need.

a.



If you move from 0 to 2 along the line you will move 2 units of distance. If you move 3 more units in the same direction, which point do you reach? What is the numeral marking this point? That's right, 5; $2 + 3 = 5$, as shown below.

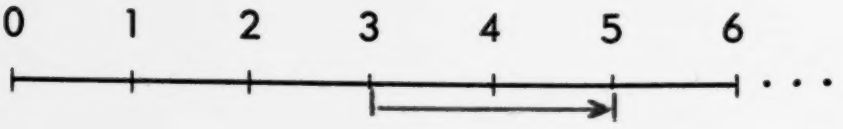
b.



Look at the line **b** above. Do you see that there are three units of distance between 2 and 5? $2 +$ **what** $= 5$? Is three units the difference between 2 and 5?

If you start at 3 and move forward 2 units, what point do you reach? It is 5 again, isn't it? $3 + 2 = 5$.

c.



Look at the line **c** above. $3 +$ **what** $= 5$? Is two units the difference between 3 and 5?

If you know the sum, 5, and one of the two addends which make it, you can find the other addend either by "taking away" or by "building up." You know that 2 and 3 make 5:

You can think either	You can think either
5 − 2 = what number?	5 − 3 = what number?
or	or
2 + what number = 5?	3 + what number = 5?

Now let's take a new look at the subtraction example which we worked on page 9 by the "take away" method.

$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{15}\cancel{6}\,{}^{15}\cancel{5} \\ 3\ 7\ 9 \\ \hline 3\ 8\ 6 \end{array}$$

Instead of thinking "15 minus 9 = what number?" you can think "9 plus what number = 15?" The answer is 6, the same as it would be by taking away.

Now in the tens' place think, "7 + what number = 15?" In the hundreds' place think, "3 + what number = 6?" When you think about building up to a number as we have just done, rather than taking away from it, you are using the **additive method** of subtraction. You are adding to find the **difference** between numbers, as you might in solving the problem that we discussed before: "I have two dollars. How much more money do I need in order to have five dollars with which to buy a sweater? Two dollars plus what equals five dollars?"

Before going any further it is important that you practice this additive method of subtraction. Try it first with SET A. You don't need to copy the examples; write just the answers. Check your thinking on page 33. Then try SET B. Turn to page 33 for SET B answers.

SET A

12	14	3	13	17	14	16	13	12	11
−4	−5	−0	−5	−8	−8	−9	−4	−5	−3

SET B

	a	b	c	d	e
1.	791	380	923	446	803
	−475	− 24	−686	−357	−168
2.	932	685	511	755	700
	−590	−598	−374	−596	−453

PART II

Keeping the Difference Constant

Boys and girls in some other countries learn a quite different way of subtracting than the way we first learn. Not only is their method of building up rather than taking away unlike ours, so also is their method of working when there are not enough ones or tens in the minuend.

Would you like to learn how to subtract in this new way? You will find that once you have mastered it, it will be very helpful both in subtraction and in division.

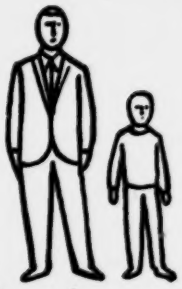
$$\begin{array}{r} {\scriptstyle 6\ 15\ 15} \\ 7\ \not{6}\ \not{5} \\ 3\ 7\ 9 \\ \hline 3\ 8\ 6 \end{array}$$

Let's take another look at the first example we used. We changed 1 ten to 10 ones to use with the 5 ones and changed 1 hundred to 10 tens to use with the 5 tens remaining. This is known as the decomposition method of subtraction because we change **one** of a larger place value in a numeral to **ten** of a smaller. That is, we break up a ten into 10 ones, a hundred into 10 tens, and so on. Look for the word **decompose** in your dictionary. Knowing the meaning of decompose will help you to understand better why this method is called decomposition.

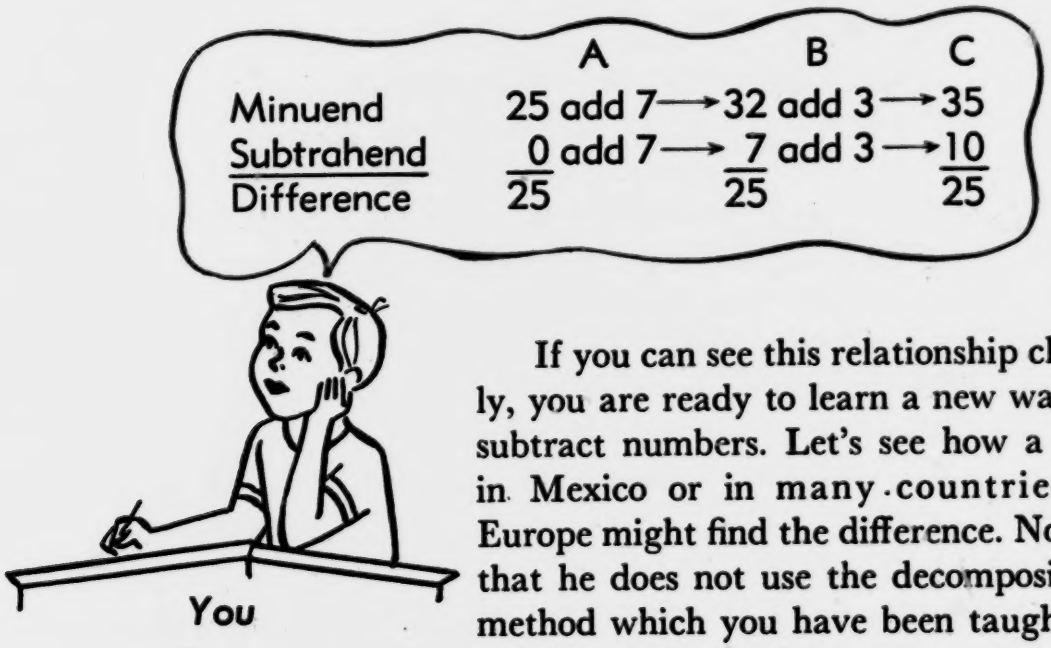
To understand the method used by people in other countries it is necessary first to understand an interesting fact about number relationships. You may already be familiar with this fact, but the way you will use it in the method we are about to learn will probably be new to you. We can illustrate this relationship with an imaginary problem involving the difference in ages between you and your father.

Suppose that your father was exactly 25 years old when you were born. The difference between your ages will of course remain 25 years throughout your lives. In other words, as each year goes by **each** of you adds a year to your age, so the difference (25 years) remains always the same, or constant.

The pictures in the sequence below might be from your family's photograph album.

A	B	C
25 yr. (Dad) 0 yr. (You) 25 yr. difference	25+7=32 yr. (Dad) 0+7= 7 yr. (You) 25 yr. difference	32+3=35 yr. (Dad) 7+3=10 yr. (You) 25 yr. difference
		
Dad and you the moment you were born	Dad and you 7 years later	Dad and you after 3 more years

Can you see from the subtraction examples in the pictures A, B, and C that when an equal amount is added to both the minuend and the subtrahend, the difference between them remains constant? Are you thinking in this way?



If you can see this relationship clearly, you are ready to learn a new way to subtract numbers. Let's see how a boy in Mexico or in many countries of Europe might find the difference. Notice that he does not use the decomposition method which you have been taught to use. As you read the explanation on the following page, point to each numeral in our example when it is mentioned.

Step 1

$$\begin{array}{r} {\scriptstyle 15} \\ 7\,6\,\not{5} \\ {\scriptstyle 8} \\ \underline{3\,\not{7}\,9} \\ 6 \end{array}$$

Step 2

$$\begin{array}{r} {\scriptstyle 16\ 15} \\ 7\,\not{6}\,\not{5} \\ {\scriptstyle 4\ 8} \\ \underline{\not{3}\,\not{7}\,9} \\ 8\,6 \end{array}$$

Step 3

$$\begin{array}{r} {\scriptstyle 16\ 15} \\ 7\,\not{6}\,\not{5} \\ {\scriptstyle 4\ 8} \\ \underline{\not{3}\,\not{7}\,9} \\ 3\,8\,6 \end{array}$$

1. I can't subtract 9 ones from 5 ones, so I add 10 ones to the 5 ones to make 15. 9 + what = 15? The answer is 6. I added 10 to the **minuend,** so in order to keep the difference constant (which is the heart of this way of subtracting) I must also add 10 to the **subtrahend.** In other words, I must compensate for (make up for) the number I added to the minuend by adding an equal number to the subtrahend. I do this by adding 1 ten in the tens' place of the subtrahend and think 8 tens instead of 7 tens.
2. I can't subtract 8 tens from 6 tens, so I add 10 tens to the 6 tens in the minuend to make 16 tens. 8 + what = 16? The answer is 8. Again I must add an equal amount to the subtrahend to keep the difference constant. The 10 tens added to the minuend equal 1 hundred, so I add 1 hundred in the hundreds' place of the subtrahend and think 4 hundreds instead of 3 hundreds.
3. 4 + what = 7? The answer is 3.

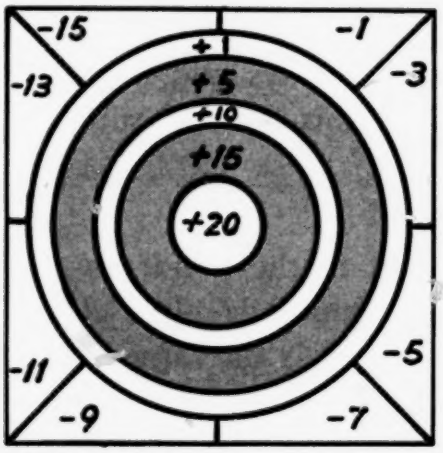
A European Boy

The method just described is known as the **equal-additions** method of subtraction because we add **equal** amounts to both the minuend and the subtrahend in order to keep the difference constant. It is also additive because we think 9 + ? = 15 rather than 15 − 9 = ?, and so on.

Study the example on this page several times until you feel that you understand the method quite well. Then turn back to SET B on page 11 and work the examples, using the additive equal-additions method. You will need pencil and paper for this. Crossing out and changing the numerals as our European friend did will help you to keep your thinking straight. Check your thinking as well as your answers on page 34.

PART III

Subtracting Several Numbers Together



Have you ever seen a dart board like this one? Hits in the circles add to your score; hits on the board outside the largest circle are minus scores and take away from the total. Mike and his sister Mary play a game every afternoon. Today Mike was ahead by a score of 178 to 148 with three more darts to throw. All three of his darts landed outside the circle, giving him scores of minus 9, minus 13, and minus 15.

Mike's Scores

+ 178
− 9
− 13
− 15

Mary found Mike's new total in this way:

178 − 9 = 169
169 − 13 = 156
156 − 15 = 141

But Mike had learned a new way to use the additive equal-additions method of subtraction. He found his new total in a way he thinks is more efficient than Mary's way. See what you think about Mike's way of working.

Step 1

```
    18
1 7 8
−   9
− 1 3 }--17
− 1 5 }
  1
  1
```

Mike

1. Since my last three scores are all minus, I can add them together and subtract their sum from 178. I start at the bottom to add in the ones' column and find that 5 + 3 + 9 = 17. 17 is greater than 8, so I add 10 ones to the 8 to make 18. (Just to keep things straight for the time being, we are placing a check mark above the numerals to indicate the "equal-additions" amounts Mike is adding to the minuend and the subtrahend). 17 + 1 = 18, so 1 belongs in the ones' place of my answer. Since I added 10 ones to the ones, I must also add 1 ten in the tens' column to keep the difference constant.

Step 2

$$\begin{array}{r} {\scriptstyle 1\,8} \\ 1\,7\,\not{8} \\ -\quad 9 \\ \hline -\ 1\,3 \\ -\ 1\,5 \\ \hline 1 \\ \hline 4\,1 \end{array} \qquad \textbf{3}$$

Step 3

$$\begin{array}{r} {\scriptstyle 1\,8} \\ 1\,7\,\not{8} \\ -\quad 9 \\ -\ 1\,3 \\ -\ 1\,5 \\ \hline 1 \\ \hline 1\,4\,1 \end{array}$$

Mike's New Score

$$\begin{array}{r} 1\,7\,8 \\ -\ 1\,5 \\ -\quad 5 \\ -\ 1\,1 \\ \hline \end{array}$$

2. Now I add in the tens' column and find that $1 + 1 + 1 = 3$ tens. $3 + 4 = 7$, so I write 4 in the tens' place of my answer. Notice that when I add the tens' column, I add in my equal-additions ten **first.** I always add from the bottom to the top in this way of subtracting.

3. I have nothing to subtract in the hundreds' column, so I write 1 in the hundreds' place of my answer. My answer of 141 is the same as Mary found by doing three separate subtractions.

Mike

Suppose that Mike's last three scores had been −15, −5, and −11. Study Mike's method carefully and use it to find his new score. Copy the example at the left and work it. You do not need to write the partial sums as shown by the boldface numerals, **17** and **3**, in Steps 1 and 2. Is your answer 147? If not, study Mike's thinking carefully, then do your work again.

The method of adding multiple (several) subtrahends and subtracting them from a single minuend can be useful. Let's see how. Suppose you have saved $17.75. You need $15.00 for camp. You want to buy a flashlight for $1.26, some toothpaste for $.32, and a compass for $1.24. Can you buy these items and still have $15.00 left?

There are several good ways to solve this problem. Let's try the method Mike used in finding his score.

Step 1

$$\begin{array}{r} {\scriptstyle 1

SET C

	a	b	c	d
1.	987	796	477	593
	−342	−231	−112	− 24
	−113	−152	− 23	−212
	−121	−212	−212	− 33
2.	848	436	642	859
	−225	−124	−214	−321
	− 32	−140	− 23	−132
	−441	−115	−121	−175

3.	7643	9785	27483	59747
	−1357	−2263	−2341	− 3815
	−2542	−2532	−6146	−24043
	−2134	−1344	−4705	− 4635
		−3122	−2291	− 3122

Check your thinking with the answers on page 36.

In all of the examples of **equal-additions subtraction** which we have worked so far, we have added only **one** ten or **one** hundred at a time to the minuend and to the subtrahend to keep the difference constant. There are times when it is necessary to add more than one ten, one hundred, and so on in order to complete the subtraction. Consider the following example with multiple subtrahends.

Step 1

$$\begin{array}{r} {\scriptstyle 2} \\ 1245 \\ -\ \ 58 \\ -296 \\ -272 \\ -187 \\ \hline 2 \end{array} \quad \text{2 3}$$

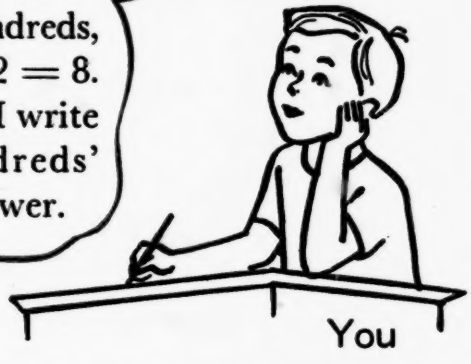
Step 2

$$\begin{array}{r} {\scriptstyle 3\,2} \\ 1245 \\ -\ \ 58 \\ -296 \\ -272 \\ -187 \\ \hline 32 \end{array} \quad \text{3 1}$$

Step 3

$$\begin{array}{r} {\scriptstyle 3\,2} \\ 1245 \\ -\ \ 58 \\ -296 \\ -272 \\ -187 \\ \hline 432 \end{array} \quad \text{8}$$

1. Adding in the ones' column I find $7 + 2 + 6 + 8 = 23$. 23 is greater than 5, so I have to add 20 to the minuend to get 25. Because $23 + 2 = 25$, I write **2** in the ones' place of my answer. I added 20 to the ones in the minuend, so I must add 2 tens to the tens' column (subtrahend) to keep the difference constant.
2. Adding the tens, I now find $2 + 8 + 7 + 9 + 5 = 31$. Since 31 is greater than 4, I add 30 and get 34 in the minuend. $31 + 3 = 34$, so I write **3** in the tens' place of my answer. I added 30 tens to the tens in the minuend, so I must add 3 hundreds (30 tens) to the hundreds' column to keep the difference constant.
3. I now add hundreds, $3 + 1 + 2 + 2 = 8$. $8 + 4 = 12$, so I write **4** in the hundreds' place of my answer.



After you have reviewed the steps above several times, do the examples in SET D. Remember that you can add to any column of the minuend the amount you need to make the subtraction possible. You must always then add an equal amount to the next column to the left in the subtrahend to keep the difference constant.

SET D

	a	b	c	d
1.	974 −244 −178 −137 −262	795 −147 −39 −292 −176	1289 −175 −139 −426 −296	2537 −739 −248 −866 −379
2.	3692 −789 −197 −996 −488	3840 −979 −825 −946 −658	2124 −365 −414 −524 −276	3482 −923 −446 −682 −834
3.	4351 −788 −371 −643 −959	9897 −1975 −1924 −1952 −2983	18754 −2595 −387 −3774 −198 −6462	23791 −2579 −369 −1428 −3747 −2689

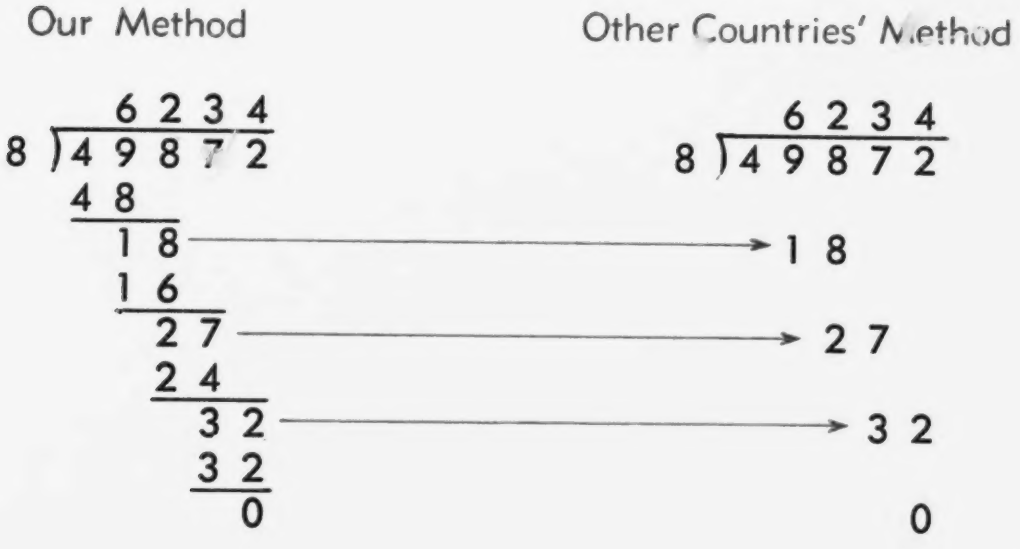
Check your thinking carefully with the answers on page 38. If you were not thinking clearly, do the examples over again and recheck them with the answers.

PART IV

Short Cuts to Subtracting in Division

If you have successfully completed the work with multiple subtrahends, you are ready to learn a new and rapid method for dividing whole numbers. You will recall that multiplication can be thought of as a short cut to addition when the addends are all the same amount. In other words, $5 \times 4 = 20$ is a short cut for $4 + 4 + 4 + 4 + 4 = 20$. We can use this knowledge and your skill with the equal-additions method of subtraction to combine the multiplication and subtraction steps in long division.

Compare the amount of written work in the two very simple examples of long division below.



Can you see that the method used in other countries omits writing the products of the multiplications? The multiplication steps and the subtraction steps are both done mentally. Only the difference and the numeral brought down from the dividend are written each time. In other words, only the **partial dividends** are written.

As we think about the following example together, point to each numeral that is mentioned in the explanation.

Step 1

```
      6
8 )4 9 8 7 2
    1 8
```

1. The first partial dividend is 49. Since $6 \times 8 = 48$, the first quotient digit is 6. Instead of writing 48, I say to myself, "48 + what = 49?" and write 1 under the 9. Next I bring down the 8. This gives me the second partial dividend, 18.

Step 2

```
      6 2
8 )4 9 8 7 2
    1 8
      2 7
```

2. Since $2 \times 8 = 16$, the second quotient digit is 2. Instead of writing 16, I say to myself, "16 + what = 18?" and I write 2 under the 8. Next I bring down the 7 for the third partial dividend.

Step 3

```
      6 2 3
8 )4 9 8 7 2
    1 8
    · 2 7
      3 2
```

3. Since $3 \times 8 = 24$, the third quotient digit is 3. Instead of writing 24, I say to myself, "24 + what = 27?" and I write 3 under the 7. Next I bring down the 2.

Step 4

```
      6 2 3 4
8 )4 9 8 7 2
    1 8
      2 7
      3 2
        0
```

4. Since $4 \times 8 = 32$, the fourth quotient digit is 4. Instead of writing 32, I say to myself, "32 + what = 32?" and I write 0 to show that there is no remainder.

You

Now copy this example:

$8\overline{)49872}$

See if you can think through the steps without referring to the booklet. If you cannot, study the steps above again before going on to SET E.

All of the examples in SET E have one-digit divisors so it will not be necessary to use equal-additions subtraction. Be sure, however, to use the additive method of subtraction. For example, in the first step of Example **1a** you would think, "9 × 6 = 54; I write 9 in the quotient. 54 + 3 = 57; I write 3 and bring down 4 to get the partial dividend 34."

SET E

	a	b	c	d
1.	6)5748	9)$51.84	7)445.9	9)7867
2.	6)20387	7)$667.52	9)23.867	8)$369.67
3.	8)$79.14	7)25969	4)$98.52	8)6096

Check your thinking with the answers on page 39.

Now that you have learned to apply this method to simple examples, let's consider a more difficult type with two digits in the divisor. It is in this type of example that you will find the additive equal-additions method of subtraction very helpful. On the following page we will work through such an example together. As you read the explanation of the steps, point to each numeral in the example at the left when it is mentioned.

Step 1

a.
$$23\overline{)759}\quad 3$$

1. a. The first partial dividend is 75. How many 23's are there in 75? Let's try 3 as the first quotient digit. We write **3** above the 5.

b.
$$23\overline{)759}\quad 3$$
$$6$$

b. Next we multiply the divisor (beginning with the ones' digit) by 3 and subtract mentally **immediately after we multiply.** $3 \times 3 = 9$. The 9 must be subtracted from the 5 of the 75, so we add 10 to the 5 to get 15. $9 +$ what $= 15$? The answer is 6; we write **6** below the 5.

c.
$$23\overline{)759}\quad 3$$
$$06$$

c. We continue to multiply: $3 \times 2 = 6$. Because we added 10 to the 5 of the 75 (the minuend), we must remember to add an equal amount to the subtrahend, so we add 1 to the 6 to get 7. (1 in the column of the 7 of the partial dividend is equivalent to 10 in the column of the 5 at its right.) $7 +$ what $= 7$? The answer is 0.

Step 2

a.
$$23\overline{)759}\quad 33$$
$$069$$

2. a. We bring down the 9 to get our next partial dividend. We can see very quickly that the next quotient digit is 3, so we write it in the correct place.

b.
$$23\overline{)759}\quad 33$$
$$069$$
$$0$$

b. Again we multiply, $3 \times 3 = 9$, and immediately subtract. $9 +$ what $= 9$? The answer is 0.

c.
$$23\overline{)759}\quad 33$$
$$069$$
$$00$$

c. Next we think, $3 \times 2 = 6$.
$6 +$ what $= 6$?
We can see that the answer to the subtraction is 0; therefore the division is completed. The zeros show that the division is "even"; that there is no remainder.

Study the explanation again. Then copy the completed example as shown in Step 2c. Beside it work the example in the usual way. Is the quotient the same? If your work is correct, it should be. The shorter method may seem difficult at first, but after you have practiced with it you will find that it speeds up your work considerably.

Here is another example to study before you do SETS F and G.

Step 1

```
  2
39)826
  04
```

1. We can see that the first partial dividend is 82. How many 39's in 82? There are 2; $2 \times 9 = 18$. We can't subtract 18 from 2. Will adding 10 to get 12 help? No, we must add 20 to get 22. 18 + what = 22? The answer is 4, so we write **4** below the 2. We continue the multiplication; $2 \times 3 = 6$. Since we must compensate for adding 20 to the minuend, we add 2 to the 6 to get 8.
   8 + what = 8? The answer is 0.

Step 2

```
  21
39)826
  046
   07
```

2. After we bring down the 6, we see that the second partial dividend is 46. How many 39's in 46? The quotient digit is 1. We multiply and subtract. 9 + what = 16? We write **7**.
   4 + what = 4? The answer is 0; therefore the quotient is 21 with a remainder of 7. We could write it as $21\frac{7}{39}$.

Think about the example again. In Step 1 why did we need 22 in the minuend? How did we get it? Why did we subtract 8 rather than 6? How did we get the 8?

In Step 2 why did we need 16? How did we get it? Why did we subtract 4 rather than 3? How did we get the 4?

Go over the steps of this example several times. When you think you have them clearly in mind, copy the example and close this booklet. If you can do the work without referring to this booklet for help, you are ready to do SET G. If you have difficulty with the steps, you may need to go back to page 21 to review the work in Part IV.

If you find that the additive subtraction itself troubles you, you may practice with SET F before doing SET G. It is up to you to decide whether or not you need the practice. The answers are on pages 41 through 46.

SET F (Subtraction practice if you need it.)

	a	b	c	d	e
1.	14+?=20	28+?=34	63+?=71	49+?=50	27+?=35
2.	48+?=51	16+?=23	27+?=32	49+?=51	28+?=33
3.	18+?=22	45+?=53	24+?=31	27+?=30	63+?=70
4.	48+?=55	36+?=44	18+?=23	49+?=53	54+?=62
5.	18+?=24	27+?=34	35+?=41	45+?=52	27+?=31
6.	72+?=80	49+?=55	48+?=50	36+?=42	18+?=26
7.	35+?=40	56+?=63	49+?=52	28+?=35	16+?=21
8.	28+?=32	64+?=71	27+?=33	36+?=40	49+?=54

SET G

	a	b	c	d
1.	$31\overline{)11998}$	$41\overline{)149864}$	$32\overline{)109952}$	$22\overline{)12474}$
2.	$84\overline{)389882}$	$83\overline{)769497}$	$26\overline{)93298}$	$57\overline{)245469}$
3.	$38\overline{)309994}$	$67\overline{)257936}$	$79\overline{)203462}$	$387\overline{)862497}$
4.	$24\overline{)99247}$	$38\overline{)265278}$	$364\overline{)169458}$	$239\overline{)85323}$

PART V

Subtracting Measures and Fractions

The equal-additions method of subtraction is based upon an important rule: **Increasing both the minuend and the subtrahend by the same amount does not change the difference between them.** Some people just call this "keeping the balance." Others prefer to say, "keeping the difference constant." Still others call it "using the principle of compensation."

Up to this point we have been concerned with adding 10 or multiples of 10 in our equal-additions. However, if you will turn back to page 13 in this booklet, you will see that when we compared your age with your father's age at two different times, we added 7 years to both the minuend and the subtrahend the first time, then added 3 more years to both the second time. Each time the difference remained constant at 25 years as we applied the principle of compensation.

In the equal-additions subtraction which you have been doing you have added first to the minuend and then made an equal addition to the subtrahend. For example, you have thought:

Minuend	$7\,^{1}6\,^{1}5$	"I add 10 ones to the 5 ones to obtain 15. I then add 1 ten to the 7 tens and get 8 tens." And so on.
Subtrahend	$\not{3}\ \not{7}\ 9$	
Difference	3 8 6	

In examples like the one above it is easier to add to the minuend first, so we do them in that way. However, there are some types of examples in which adding **first to the subtrahend** makes a short cut possible, for instance, when we subtract different units of measures or common fractions. You can add to the subtrahend first rather than adding first to the minuend, so long as you add an equal amount to both, thus **keeping the difference constant.** Let's see how the principle of compensation operates in these cases.

Mike and Mary, whom we have met before, want to find the difference between their ages. Mike is 12 years and 3 months old. Mary is 9 years and 7 months old. Mary finds the difference in the usual way. Let's look at her written work on the left and follow her thinking below.

a		b
12 yr. 3 mo.	=	11 yr. 15 mo.
−9 yr. 7 mo.	=	−9 yr. 7 mo.
		2 yr. 8 mo.

Mary

a. I can't subtract 7 months from 3 months, so I take 1 year from the 12 years and change it to 12 months, which I add to the 3 months to get 15 months. 12 years and 3 months **equals** 11 years and 15 months.

b. Now, with the minuend written as 11 years and 15 months, I can subtract: $15 - 7 = 8$ and $11 - 9 = 2$.

Mike again put his knowledge of equal-additions to work and solved his problem entirely in his head. Let's follow Mike's thinking. (Remember that the work is only in Mike's thinking and not on his paper.)

a. 12 yr. 3 mo.
−9 yr. 7 mo.

b. 12 yr. 8 mo.
−10 yr. 0 mo.
2 yr. 8 mo.

a. Since I can't subtract 7 months from 3 months I'll add 5 months to the 7 months (equals 1 yr.) to do away with months in the subtrahend.

b. Now the subtrahend is 10 years. To keep the difference constant I must also add 5 months to the 3 months in the minuend. 3 months + 5 months = 8 months. I can subtract (by the additive method): $0 + 8 = 8$ and $10 + 2 = 12$.

Mike

Did you notice that Mike made his first addition in the subtrahend? This was done to clear the subtrahend of the part of a year (7 mo.). Mike does not use equal signs with his method as Mary did with hers, because 12 yr. 3 mo. **does not** equal 12 yr. 8 mo. and 10 yr. **does not** equal 9 yr. 7 mo.

The method that Mike used works equally well with other measures. Here is an example with yards and feet.

a. Minuend 9 yd. $1\frac{1}{2}$ ft.
Subtrahend 4 yd. 2 ft.

a. What can be done to clear the subtrahend of the part of a yard? (3 ft. = 1 yd.)

b. Minuend 9 yd. $2\frac{1}{2}$ ft.
Subtrahend 5 yd.
Difference 4 yd. $2\frac{1}{2}$ ft.

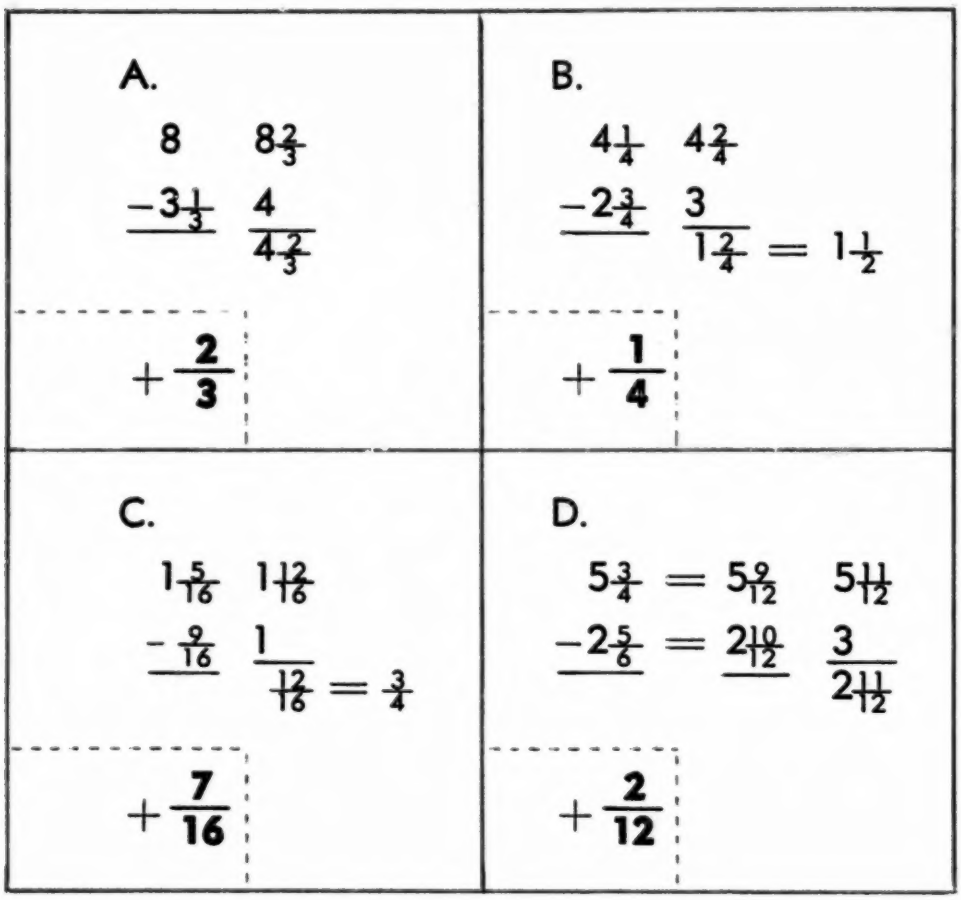
b. Was 1 ft. added to both the subtrahend and the minuend?

Now do the examples in SET H. Do not copy the examples. Write only the answers. If you find this difficult, review Mike's example and that above before going on. You may want to find a table of measures in your arithmetic textbook. Check your thinking on page 47.

SET H

	a	b	c
1.	7 lb. 3 oz. − 5 lb. 12 oz.	11 ft. 4 in. − 3 ft. 7 in.	12 yr. − 8 yr. 3 mo.
2.	25 yd. 6 in. − 11 yd. 17 in.	9T. 400 lb. − 5T. 900 lb.	4 gal. 1 qt. − 3 gal. 3 qt.
3.	8 min. 6.25 sec. − 3 min. 43.50 sec.	6 hr. 28 min. − 2 hr. 45 min.	9 lb. $4\frac{1}{2}$ oz. − 5 lb. $8\frac{1}{2}$ oz.

This same procedure works well with common fractions. Let's look at several examples.



Notice that in each of these examples the subtrahend has been changed to a whole number by adding the amount which is shown in the small box in the lower left-hand corner of each frame. The same amount has been added to the minuend to keep the difference constant. Notice that equal signs were used only in Frame D when changing to a common denominator. $5\frac{3}{4}$ is equal to $5\frac{9}{12}$, but it would certainly be wrong to say $5\frac{9}{12} = 5\frac{11}{12}$. **Equal signs should be used only when two quantities are truly equal to each other.**

If you really understand the equal-additions method and the principle of compensation, you should be able to do most of the exampies in SET I mentally. Try thinking through each example, and write only the answer. In the case of fractions

with unlike denominators you may need to write the part where you change to common denominators. Check your answers with those on page 47.

SET I

	a	b	c	d	e	f	g	h
1.	$5\frac{3}{8}$	$6\frac{7}{10}$	$3\frac{5}{12}$	$7\frac{3}{5}$	$1\frac{5}{16}$	$4\frac{1}{10}$	$7\frac{9}{32}$	4
	$-2\frac{5}{8}$	$-3\frac{9}{10}$	$-2\frac{11}{12}$	$-4\frac{4}{5}$	$-\frac{9}{16}$	$-1\frac{7}{10}$	$-5\frac{13}{32}$	$-3\frac{7}{10}$
2.	$4\frac{1}{2}$	$6\frac{2}{3}$	$9\frac{2}{3}$	$7\frac{5}{12}$	$4\frac{3}{5}$	$4\frac{5}{32}$	$7\frac{1}{2}$	$5\frac{5}{6}$
	$-1\frac{3}{4}$	$-4\frac{5}{6}$	$-3\frac{3}{4}$	$-2\frac{1}{2}$	$-3\frac{9}{10}$	$-2\frac{3}{4}$	$-4\frac{9}{16}$	$-2\frac{7}{8}$

Do the ways of working which you have learned seem like magic to some of your friends? They are not magic, as you know, but are based on the use of relationships among numbers. As you increase your understanding of such relationships and your skill in using them, you will find that your speed, efficiency, and pleasure in arithmetic increase accordingly. In another Independent Learning booklet, **The Decimal Point in Division,** you will gain further understanding of number relationships.

CHECKING YOUR THINKING

In this section you will find the answers to the examples and also some help in learning the new ways of working presented in this booklet. Notice that "Your Thinking" is described as the necessary steps you perform in arriving at your answer. To help you "see" your thinking, some numerals are printed in a different kind or color of type.

Look at the answers to SET A on the opposite page.

Your first example in SET A is at the left. The way you arrive at your answer is by thinking, "4 + what number = 12?" We show this in the bottom line of your answer by the different order and color of numerals: **4** + **8** = **12**. The **8** is the number which you **think** as you add to find the difference and which you **write** as your answer.

$$\begin{array}{r} 12 \\ -\,4 \\ \hline \end{array}$$

Now look at the answers to SET B, Additive Equal-Additions Method, on page 34. The original example you worked on page 11 is shown at the left. In "Your Thinking (working from ones' to hundreds' column)" these numerals stay the same. But the numbers you **think** as you add to find the difference and which you **write** in your answer are shown (in Example 1 **a**) as **6**, as **1**, and as **3**. Also, because you are using the equal-additions method, the numbers you add to the minuend and to the subtrahend are shown as **1** under ones' column and then as **1** under tens' column.

$$\begin{array}{r} 791 \\ -\,475 \\ \hline \end{array}$$

PART I

SET A Additive Method

12	14	3	13	17
−4	−5	−0	−5	−8
4+8=12	**5+9=14**	**0+3=3**	**5+8=13**	**8+9=17**

Your Thinking

14	16	13	12	11
−8	−9	−4	−5	−3
8+6=14	**9+7=16**	**4+9=13**	**5+7=12**	**3+8=11**

Your Thinking

Did you remember to **build up** rather than **take away**?

Did you think 4 **plus** what number = 12? instead of 12 **minus** 4 = 8?

If you find it difficult to think in this way, draw a number line with 18 or 20 spaces. Use it to figure out the subtraction in SET A. Start with the subtrahend and build up to the minuend. For example, for 12 − 4 locate 4 on the number line and count the units of distance to reach 12.

You will have more practice in the additive method in SET B.

SET B Additive Method

	a	b	c	d	e
1.	316	356	237	89	635
2.	342	87	137	159	247

In Example **1a**, 791 − 475, did you think:

5 + 6 = 11, 7 + 1 = 8, and 4 + 3 = 7?

Be sure that you can work quickly and accurately with the additive method before going on in this booklet. If you need a little more practice, find a set of subtraction examples in your arithmetic textbook. Use the additive method to do the examples. Again, do as much of the work as you can in your head and write only the answers.

PART II

SET B Additive Equal-Additions Method
Your Thinking (working from ones' to hundreds' column)

	Your Answer	Hundreds' Column	Tens' Column	*START HERE* Ones' Column
1. a.	316	4+**3**=7; write **3**	1+7+**1**=9; write **1**	5+**6**=11; write **6**
b.	356	Bring down **3**	1+2+**5**=8; write **5**	4+**6**=10; write **6**
c.	237	1+6+**2**=9; write **2**	1+8+**3**=12; write **3**	6+**7**=13; write **7**
d.	89	1+3+0=4; think 0	1+5+**8**=14; write **8**	7+**9**=16; write **9**
e.	635	1+1+**6**=8; write **6**	1+6+**3**=10; write **3**	8+**5**=13; write **5**
2. a.	342	1+5+**3**=9; write **3**	9+**4**=13 write **4**	0+**2**=2; write **2**
b.	87	1+5+0=6; think 0	1+9+**8**=18; write **8**	8+**7**=15; write **7**
c.	137	1+3+**1**=5; write **1**	1+7+**3**=11; write **3**	4+**7**=11; write **7**
d.	159	1+5+**1**=7; write **1**	1+9+**5**=15; write **5**	6+**9**=15; write **9**
e.	247	1+4+**2**=7; write **2**	1+5+**4**=10; write **4**	3+**7**=10; write **7**

After you have completed this set of examples, find another set in your arithmetic textbook. This time practice the **equal-additions** method without crossing out any numbers. Write only the answers. Check your results by adding the **difference** to the **subtrahend.** The sum should equal the **minuend.**

Example:

$$\begin{array}{r} 765 \\ -379 \\ \hline 386 \\ \hline\hline 765 \end{array}$$

The sum of 379 and 386 is 765, which is equal to the minuend. You may like to check in this way:

Say, "6 and 9 are 15; check: $76\overset{\checkmark}{5}$;

$1 + 8 + 7 = 16$; check: $7\overset{\checkmark\checkmark}{6}5$;

$1 + 3 + 3 = 7$; check: $\overset{\checkmark\checkmark\checkmark}{7}65$."

You should be able to use the **additive equal-additions method** with speed and accuracy before going on to Part III.

PART III

SET C Multiple Subtrahends

Your Thinking (working from ones' to thousands' column)

	Your Answer	Thousands' Column	Hundreds' Column	Tens' Column	*START HERE* Ones' Column
1. a.	411		1+1+3=5; 5+**4**=9; write **4**	2+1+4=7; 7+**1**=8; write **1**	1+3+2=6; 6+**1**=7; write **1**
b.	201		2+1+2=5; 5+**2**=7; write **2**	1+5+3=9; 9+**0**=9; write **0**	2+2+1=5; 5+**1**=6; write **1**
c.	130		2+1=3; 3+**1**=4; write **1**	1+2+1=4; 4+**3**=7; write **3**	2+3+2=7; 7+**0**=7; write **0**
d.	324		2+**3**=5; write **3**	1+3+1+2=7; 7+**2**=9; write **2**	3+2+4=9; 9+**4**=13; write **4**
2. a.	150		1+4+2=7; 7+**1**=8; write **1**	4+3+2=9; 9+**5**=14; write **5**	1+2+5=8; 8+**0**=8; write **0**
b.	57		1+1+1+1=4; 4+0=4; think 0	1+1+4+2=8; 8+**5**=13; write **5**	5+0+4=9; 9+**7**=16; write **7**
c.	284		1+1+2=4; 4+**2**=6; write **2**	1+2+2+1=6; 6+**8**=14; write **8**	1+3+4=8; 8+**4**=12; write **4**

d.	231		1+1+1+3=6; 6+**2**=8; write **2**	7+3+2=12; 12+**3**=15; write **3**	5+2+1=8; 8+**1**=9; write **1**
3. a.	1,610	1+2+2+1=6; 6+**1**=7; write **1**	1+1+5+3=10; 10+**6**=16; write **6**	1+3+4+5=13; 13+**1**=14; write **1**	4+2+7=13; 13+**0**=13; write **0**
b.	524	1+3+1+2+2=9; 9+0=9; think 0	1+1+3+5+2=12; 12+**5**=17; write **5**	1+2+4+3+6=16; 16+**2**=18; write **2**	2+4+2+3=11; 11+**4**=15; write **4**
c.	12,000	1+2+4+6+2=15; 15+**12**=27; write **12**	1+2+7+1+3=14; 14+**0**=14; write **0**	1+9+0+4+4=18; 18+**0**=18; write **0**	1+5+6+1=13; 13+**0**=13; write **0**
d.	24,132	1+3+4+24+3=35; 35+**24**=59; write **24**	1+1+6+0+8=16; 16+**1**=17; write **1**	1+2+3+4+1=11; 11+**3**=14; write **3**	2+5+3+5=15; 15+**2**=17; write **2**

Did you notice a shortcut in Examples **3a** and **3d**? Did you think of this when you were doing your examples? The last section of Part III introduces more difficult examples of multiple subtrahends. If you had difficulty, do SET C again before turning to the last section of Part III.

SET D Multiple Subtrahends

Your Thinking (working from ones' to thousands' column)

	Your Answer	Thousands' Column	Hundreds' Column	Tens' Column	*START HERE* Ones' Column
1. a.	153		2+6+**1**=9; write **1**	2+20+**5**=27; write **5**	21+**3**=24; write **3**
b.	141		2+4+**1**=7; write **1**	2+23+**4**=29; write **4**	24+**1**=25; write **1**
c.	253		2+8+**2**=12; write **2**	2+21+**5**=28; write **5**	26+**3**=29; write **3**
d.	305		2+20+**3**=25; write **3**	3+20+**0**=23; write **0**	32+**5**=37; write **5**
2. a.	1,222		3+21+**12**=36; write **12**	3+34+**2**=39; write **2**	30+**2**=32; write **2**
b.	432		2+32+**4**=38; write **4**	3+18+**3**=24; write **3**	28+**2**=30; write **2**
c.	545		2+14+**5**=21; write **5**	2+16+**4**=22; write **4**	19+**5**=24; write **5**
d.	597		2+27+**5**=34; write **5**	2+17+**9**=28; write **9**	15+**7**=22; write **7**
3. a.	1,590		3+25+**15**=43; write **15**	2+24+**9**=35; write **9**	21+**0**=21; write **0**
b.	1,063	3+5+**1**=9; write **1**	2+36+**0**=38; write **0**	1+22+**6**=29; write **6**	14+**3**=17; write **3**
c.	5,338	2+11+**5**=18; write **5**	4+20+**3**=27; write **3**	3+39+**3**=45; write **3**	26+**8**=34; write **8**
d.	12,979	3+8+**12**=23; write **12**	3+25+**9**=37; write **9**	5+27+**7**=39; write **7**	42+**9**=51; write **9**

PART IV

SET E Division

Your Thinking (working from left to right)

	Your Answer	Step 1	Step 2	Step 3	Step 4
a.	958	9×6=54; 54+**3**=57; write **3**; bring down 4	5×6=30; 30+**4**=34; write **4**; bring down 8	8×6=48; 48+**0**=48; write **0**	
b.	$5.76	5×9=45; 45+**6**=51; write **6**; bring down 8	7×9=63; 63+**5**=68; write **5**; bring down 4	6×9=54; 54+**0**=54; write **0**	
c.	63.7	6×7=42; 42+**2**=44; write **2**; bring down 5	3×7=21; 21+**4**=25; write **4**; bring down 9	7×7=49; 49+**0**=49; write **0**	
d.	874 R1	8×9=72; 72+**6**=78; write **6**; bring down 6	7×9=63; 63+**3**=66; write **3**; bring down 7	4×9=36; 36+**1**=37; write **1** (remainder)	
a.	3397 R5	3×6=18; 18+**2**=20; write **2**; bring down 3	3×6=18; 18+**5**=23; write **5**; bring down 8	9×6=54; 54+**4**=58; write **4**; bring down 7	7×6=42; 42+**5**=47; write **5** (remainder)
b.	$95.36	9×7=63; 63+**3**=66; write **3**; bring down 7	5×7=35; 35+**2**=37; write **2**; bring down 5	3×7=21; 21+**4**=25; write **4**; bring down 2	6×7=42; 42+**0**=42; write **0**

SET E Division

Your Thinking (working from left to right)

	Your Answer	Step 1	Step 2	Step 3	Step 4
2. c.	2.651 R8	$2\times9=18$; $18+\mathbf{5}=23$; write **5**; bring down 8	$6\times9=54$; $54+\mathbf{4}=58$; write **4**; bring down 6	$5\times9=45$; $45+\mathbf{1}=46$; write **1**; bring down 7	$1\times9=9$; $9+\mathbf{8}=17$; write **8** (remainder)
d.	$46.20 R7¢	$4\times8=32$; $32+\mathbf{4}=36$; write **4**; bring down 9	$6\times8=48$; $48+\mathbf{1}=49$; write **1**; bring down 6	$2\times8=16$; $16+\mathbf{0}=16$; write **0**; bring down 7	$7\div8=0$ write **7** (remainder)
3. a.	$9.89 R2¢	$9\times8=72$; $72+\mathbf{7}=79$; write **7**; bring down 1	$8\times8=64$; $64+\mathbf{7}=71$; write **7**; bring down 4	$9\times8=72$; $72+\mathbf{2}=74$; write **2** (remainder)	
b.	3709 R6	$3\times7=21$; $21+\mathbf{4}=25$; write **4**; bring down 9	$7\times7=49$; $49+\mathbf{0}=49$; write **0**; bring down 6	6 is smaller than 7; bring down 9	$9\times7=63$; $63+\mathbf{6}=69$; write **6** (remainder)
c.	$24.63	$2\times4=8$; $8+\mathbf{1}=9$; write **1**; bring down 8	$4\times4=16$; $16+\mathbf{2}=18$; write **2**; bring down 5	$6\times4=24$; $24+\mathbf{1}=25$; write **1**; bring down 2	$3\times4=12$; $12+\mathbf{0}=12$; write **0**
d.	76	$7\times8=56$; $56+\mathbf{4}=60$; write **4**; bring down 9	$6\times8=48$; $48+\mathbf{1}=49$; write **1**; bring down 6	$2\times8=16$; $16+\mathbf{0}=16$; write **0**	

SET F (Subtraction practice if you need it.)

	a	b	c	d	e
1.	6	6	8	1	8
2.	3	7	5	2	5
3.	4	8	7	3	7
4.	7	8	5	4	8
5.	6	7	6	7	4
6.	8	6	2	6	8
7.	5	7	3	7	5
8.	4	7	6	4	5

Because the examples in this set are more difficult, all of the written work has been printed on the left side of the page as it should appear on your paper. The steps in your thinking have been printed on the right side of the page. Notice that the **boldface** numerals are those you write in the example after you have subtracted mentally; the numerals shown in color are the equal-addition numerals you carry in your head.

The answers and your thinking begin on the following page.

SET G Division

1. a.

$$31\overline{)11998}\quad 387$$

269
218
01R

Step 1: 3×1=3; 3+**6**=9 — 3×3=9; 9+**2**=11
Step 2: 8×1=8; 8+**1**=9 — 8×3=24; 24+**2**=26
Step 3: 7×1=7; 7+**1**=8 — 7×3=21; 21+**0**=21

b.

$$41\overline{)149864}\quad 3655$$

268
226
214
09R

Step 1: 3×1=3; 3+**6**=9 — 3×4=12; 12+**2**=14
Step 2: 6×1=6; 6+**2**=8 — 6×4=24; 24+**2**=26
Step 3: 5×1=5; 5+**1**=6 — 5×4=20; 20+**2**=22
Step 4: 5×1=5; 5+**9**=**14** — 5×4+**1**=21; 21+**0**=21

c.

$$32\overline{)109952}\quad 3436$$

139
115
192
00

Step 1: 3×2=6; 6+**3**=9 — 3×3=9; 9+**1**=10
Step 2: 4×2=8; 8+**1**=9 — 4×3=12; 12+**1**=13
Step 3: 3×2=6; 6+**9**=**15** — 3×3+**1**=10; 10+**1**=11
Step 4: 6×2=12; 12+**0**=12 — 6×3+**1**=19; 19+**0**=19

d.

$$22\overline{)12474}\quad 567$$

147
154
00

Step 1: 5×2=10; 10+**4**=**14** — 5×2+**1**=11; 11+**1**=12
Step 2: 6×2=12; 12+**5**=**17** — 6×2+**1**=13; 13+**1**=14
Step 3: 7×2=14; 14+**0**=14 — 7×2+**1**=15; 15+**0**=15

2. a.
$$\begin{array}{r} 4641 \\ 84\overline{)389882} \\ 538 \\ 348 \\ 122 \\ 38\text{R} \end{array}$$

Step 1: $4\times4=16;\ 16+\mathbf{3}=19$ — $4\times8+\mathbf{1}=33;\ 33+\mathbf{5}=38$
Step 2: $6\times4=24;\ 24+\mathbf{4}=28$ — $6\times8+\mathbf{2}=50;\ 50+\mathbf{3}=53$
Step 3: $4\times4=16;\ 16+\mathbf{2}=18$ — $4\times8+\mathbf{1}=33;\ 33+\mathbf{1}=34$
Step 4: $1\times4=4;\ 4+\mathbf{8}=12$ — $1\times8+\mathbf{1}=9;\ 9+\mathbf{3}=12$

b.
$$\begin{array}{r} 9271 \\ 83\overline{)769497} \\ 224 \\ 589 \\ 087 \\ 4\text{R} \end{array}$$

Step 1: $9\times3=27;\ 27+\mathbf{2}=29$ — $9\times8+\mathbf{2}=74;\ 74+\mathbf{2}=76$
Step 2: $2\times3=6;\ 6+\mathbf{8}=14$ — $2\times8+\mathbf{1}=17;\ 17+\mathbf{5}=22$
Step 3: $7\times3=21;\ 21+\mathbf{8}=29$ — $7\times8+\mathbf{2}=58;\ 58+\mathbf{0}=58$
Step 4: $1\times83=83;\ 83+\mathbf{4}=87$

c.
$$\begin{array}{r} 3588 \\ 26\overline{)93298} \\ 152 \\ 229 \\ 218 \\ 10\text{R} \end{array}$$

Step 1: $3\times6=18;\ 18+\mathbf{5}=23$ — $3\times2+\mathbf{2}=8;\ 8+\mathbf{1}=9$
Step 2: $5\times6=30;\ 30+\mathbf{2}=32$ — $5\times2+\mathbf{3}=13;\ 13+\mathbf{2}=15$
Step 3: $8\times6=48;\ 48+\mathbf{1}=49$ — $8\times2+\mathbf{4}=20;\ 20+\mathbf{2}=22$
Step 4: $8\times6=48;\ 48+\mathbf{0}=48$ — $8\times2+\mathbf{4}=20;\ 20+\mathbf{1}=21$

d.
$$\begin{array}{r} 4306 \\ 57\overline{)245469} \\ 174 \\ 0369 \\ 27\text{R} \end{array}$$

Step 1: $4\times7=28;\ 28+\mathbf{7}=35$ — $4\times5+\mathbf{3}=23;\ 23+\mathbf{1}=24$
Step 2: $3\times7=21;\ 21+\mathbf{3}=24$ — $3\times5+\mathbf{2}=17;\ 17+\mathbf{0}=17$
Step 3: $36\div57=0$
Step 4: $6\times7=42;\ 42+\mathbf{7}=49$ — $6\times5+\mathbf{4}=34;\ 34+\mathbf{2}=36$

SET G Division

3. a.
```
      8157
38)309994
   059
    219
     294
      28R
```

Step 1: 8×8=64; 64+**5**=69 — 8×3+6=30; 30+**0**=30
Step 2: 1×38=38; 38+**21**=59
Step 3: 5×8=40; 40+**9**=49 — 5×3+4=19; 19+**2**=21
Step 4: 7×8=56; 56+**8**=64 — 7×3+6=27; 27+**2**=29

b.
```
      3849
67)257936
   569
    333
     656
      53R
```

Step 1: 3×7=21; 21+**6**=27 — 3×6+2=20; 20+**5**=25
Step 2: 8×7=56; 56+**3**=59 — 8×6+5=53; 53+**3**=56
Step 3: 4×7=28; 28+**5**=33 — 4×6+3=27; 27+**6**=33
Step 4: 9×7=63; 63+**3**=66 — 9×6+6=60; 60+**5**=65

c.
```
      2575
79)203462
   454
    596
     432
      37R
```

Step 1: 2×9=18; 18+**5**=23 — 2×7+2=16; 16+**4**=20
Step 2: 5×9=45; 45+**9**=54 — 5×7+5=40; 40+**5**=45
Step 3: 7×9=63; 63+**3**=66 — 7×7+6=55; 55+**4**=59
Step 4: 5×9=45; 45+**7**=52 — 5×7+5=40; 40+**3**=43

d.
```
      2228
387)862497
   0884
    1109
     3357
      261R
```

Step 1: $2\times7=14$; $14+\mathbf{8}=22$ — $2\times8+2=18$; $18+\mathbf{8}=26$
$2\times3+2=8$; $8+\mathbf{0}=8$
Step 2: $2\times7=14$; $14+\mathbf{0}=14$ — $2\times8+1=17$; $17+\mathbf{1}=18$
$2\times3+1=7$; $7+\mathbf{1}=8$
Step 3: $2\times7=14$; $14+\mathbf{5}=19$ — $2\times8+1=17$; $17+\mathbf{3}=20$
$2\times3+2=8$; $8+\mathbf{3}=11$
Step 4: $8\times7=56$; $56+\mathbf{1}=57$ — $8\times8+5=69$; $69+\mathbf{6}=75$
$8\times3+7=31$; $31+\mathbf{2}=33$

4. a.
```
      4135
24)99247
   032
    84
     127
      07R
```

Step 1: $4\times4=16$; $16+\mathbf{3}=19$ — $4\times2+1=9$; $9+\mathbf{0}=9$
Step 2: $1\times24=24$; $24+\mathbf{8}=32$
Step 3: $3\times4=12$; $12+\mathbf{2}=14$ — $3\times2+1=7$; $7+\mathbf{1}=8$
Step 4: $5\times4=20$; $20+\mathbf{7}=27$ — $5\times2+2=12$; $12+\mathbf{0}=12$

b.
```
      6981
38)265278
   372
    307
     038
      0
```

Step 1: $6\times8=48$; $48+\mathbf{7}=55$ — $6\times3+5=23$; $23+\mathbf{3}=26$
Step 2: $9\times8=72$; $72+\mathbf{0}=72$ — $9\times3+7=34$; $34+\mathbf{3}=37$
Step 3: $8\times8=64$; $64+\mathbf{3}=67$ — $8\times3+6=30$; $30+\mathbf{0}=30$
Step 4: $1\times38=38$; $38+\mathbf{0}=38$

SET G Division

4. c.
$$364\overline{)169458}\quad 465$$
2385
2018
198R

Step 1: $4\times4=16;\ 16+\mathbf{8}=24$
$4\times3+\mathbf{2}=14;\ 14+\mathbf{2}=16$
Step 2: $6\times4=24;\ 24+\mathbf{1}=25$
$6\times3+\mathbf{3}=21;\ 21+\mathbf{2}=23$
Step 3: $5\times4=20;\ 20+\mathbf{8}=28$
$5\times3+\mathbf{4}=19;\ 19+\mathbf{1}=20$

$4\times6+\mathbf{2}=26;\ 26+\mathbf{3}=29$

$6\times6+\mathbf{2}=38;\ 38+\mathbf{0}=38$

$5\times6+\mathbf{2}=32;\ 32+\mathbf{9}=41$

d.
$$239\overline{)85323}\quad 357$$
1362
1673
000

Step 1: $3\times9=27;\ 27+\mathbf{6}=33$
$3\times2+\mathbf{1}=7;\ 7+\mathbf{1}=8$
Step 2: $5\times9=45;\ 45+\mathbf{7}=52$
$5\times2+\mathbf{2}=12;\ 12+\mathbf{1}=13$
Step 3: $7\times9=63;\ 63+\mathbf{0}=63$
$7\times2+\mathbf{2}=16;\ 16+\mathbf{0}=16$

$3\times3+\mathbf{3}=12;\ 12+\mathbf{3}=15$

$5\times3+\mathbf{5}=20;\ 20+\mathbf{6}=26$

$7\times3+\mathbf{6}=27;\ 27+\mathbf{0}=27$

PART V

SET H Measures:

	a	b	c
1.	1 lb. 7 oz.	7 ft. 9 in.	3 yr. 9 mo.
2.	13 yd. 25 in.	3 T. 1500 lb.	2 qt.
3.	4 min. 22.75 sec.	3 hr. 43 min.	3 lb. 12 oz.

SET I Common Fractions:

	a	b	c	d	e	f	g	h
1.	$2\frac{3}{4}$	$2\frac{4}{5}$	$\frac{1}{2}$	$2\frac{4}{5}$	$\frac{3}{4}$	$2\frac{2}{5}$	$1\frac{7}{8}$	$\frac{3}{10}$
2.	$2\frac{3}{4}$	$1\frac{5}{6}$	$5\frac{11}{12}$	$4\frac{11}{12}$	$\frac{7}{10}$	$1\frac{13}{32}$	$2\frac{15}{16}$	$2\frac{23}{24}$

This Independent Learning Booklet is part of an experimental project to provide mathematics materials for able pupils and to develop a mathematics program adapted to individual differences. The project is based on several studies and exploratory programs carried on in San Diego County during the past several years, such as those sponsored by the San Diego Metropolitan Education Study Council and the Assistant Superintendents Group. Objectives of this phase of the project are to stimulate and widen pupil interest in various areas of mathematics; to develop deeper insights into mathematical principles, concepts, and relationships; to introduce aspects of modern mathematical thinking; and to develop motivation for able pupils to continue to study and explore mathematics and related fields.

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